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A NOTE ON THE CONTINUOUS DIFFERENTIABILITY OF AN EXPECTED UTILI--ETC(U)

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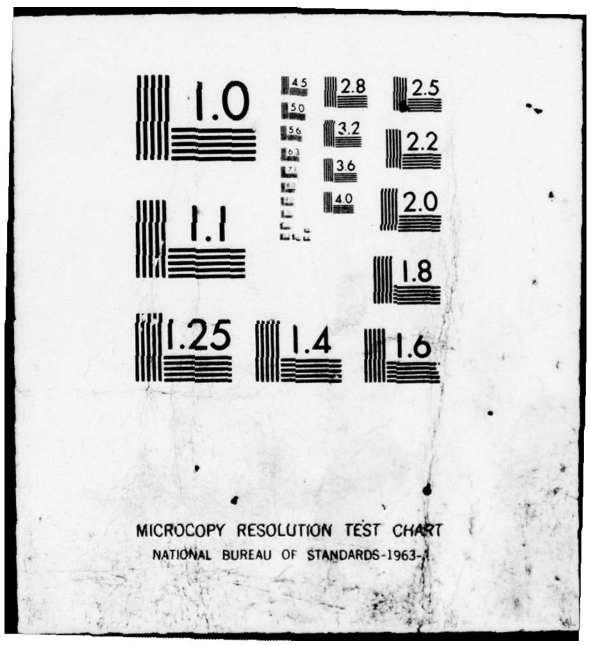
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A NOTE ON THE CONTINUOUS DIFFERENTIABILITY OF AN EXPECTED UTILITY FUNCTION: A TWO PERIODS CONSUMER DECISION PROBLEM *

by

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A NOTE ON THE CONTINUOUS DIFFERENTIABILITY OF AN EXPECTED UTILITY FUNCTION: A TWO PERIODS CONSUMER DECISION PROBLEM

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In this note, we consider an intertemporal consumer decision problem involving two periods--current and future, linked by the expectations about environment. A similar formulation has been extensively studied for the existence of "temporary equilibrium" a la Hicks (1946) (see for instance the works of Stigum (1969), Arrow and Hahn (1971), Grandmont (1974), Sondermann (1974) and Green (1973), among others). In the above works, using a standard dynamic programming technique, the indirect or expected utility function is defined as a utility index over actions, incorporated with the expectations of the future environment. The continuity of an expected utility function follows from the continuity of a monotonic and concave von Neumann-Morgenstern utility function on consequences (and hence, among other conditions, the upper hemi-continuity of the plan for future consumption), and the continuity of an expectation function on current and past observations of the economic data, respectively (see Grandmont (1972), (1974), Sondermann (1974), Delbaen (1974), and also Jordan (1977) for a case of many periods). Toward the studies of comparative statics and dynamics of the system, however, the bare continuity may not be sufficient for the analysis. We need continuous differentiability of the functions which describe the consumer's characteristics. More recent works on approximations theorems (for example, see Grodal (1974), Kannai (1974), and Mas-Collel (1974)) give further justification to the use of differentiable approach in economics. Unfortunately, in the framework of

temporary equilibrium theory, the continuous differentiability of an expected utility function may not be simply implied by the sufficient differentiability of a primitive von Neumann-Morgenstern utility and an expectation function, respectively. In this note, additional classical assumptions on the utility or preference and expectation are imposed so that the consumption plan made for the future period exhibits the sufficient differentiability requirement, which in turn restates the differentiability of the expected utility function with respect to all variables. It is believed that the technique and result presented here will provide foundations for the studies of comparative statics and dynamics of temporary equilibrium theory (see for instance Fuchs (1976), Fuchs and Laroque (1976), Kalman and Lin (1978), and Lin (1977)).

In the following, we shall adopt a simple temporary equilibrium model as modified by Grandmont in (1974) for the study of a money economy. The reader is referred to Grandmont (1974) and Sondermann (1974) among others for more details.

Suppose there are two periods t and t+1. For each period, there are ℓ commodities.[1] Let $P = \{z \in R^{\ell} : z \gg^{2} 0\}$ be the consumption space in which an element x^t is a consumption bundle for the consumer at time t. We assume that all ℓ commodities are perishable and have to be consumed during one period. Let $R_+ = \{z \in R : z \geq 0\}$ and $m^t \in R_+$ is the consumer's money holding at time t. We note that money is a unit of account and can be stored at no cost between periods t and t+1. Let $S = \{s = (p,1) \in R^{\ell+1} : p \gg 0\}$ be the monetary price space, where the price of money is given as unity and p is the price

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[1] This can be generalized in a straightforward way to a different number of commodities in periods t and t+1, respectively.

[2] If $x, x' \in R^{\ell}$, $x \geq x'$ means $x_i \geq x_i'$ for all i; $x > x'$ means $x \geq x'$ and $x \neq x'$; $x \gg x'$ means $x_i > x_i'$ for all i.

system of ℓ commodities in terms of money. At the beginning of period t, the consumer knows with certainty his initial commodity-money endowment, i.e., $(x^t, m^{t-1}) \epsilon P \times R_+$, where the money endowment m^{t-1} is the cash balance carried over from the previous period. Then an <u>action</u> at time t is defined by a commodity-money holding, or $(x^t, m^t) \epsilon P \times R_+$. A possible <u>consequence</u> of an action is a pair of current and future consumptions denoted by $(x^t, x^{t+1}) \epsilon P \times P$.

To specify a subjective uncertainty about the future environment, one assume that the consumer forecasts future prices and commodity endowments which take forms of probability distributions on S x P. In general, this forecast will depend on the current and past information available in the model. For simplification, such an anticipation is assumed to rely upon the currently quoted price system s^t only. Let $(S \times P, \mathcal{B}(S \times P))$ be a measurable space with $\mathcal{B}(S \times P)$ denoting the Borel σ - field of S x P, and the agent's expectation be a real-valued function γ defined on $S \times \mathcal{B}(S \times P)$. Let $\mathcal{M}(S \times P)$ be the family of probability measures defined on the measurable space (S x P, $\mathcal{B}(S \times P)$), we assume further that $\gamma(s^t; \cdot) \epsilon \mathcal{M}(S \times P)$ for all $s^t \epsilon S$. In particular, for $B \epsilon \mathcal{B}(S \times P)$, $\gamma(s^t; B)$ is the probability of B if s^t is quoted on the t-th market. We have the following assumption:

<u>Assumption 1</u>: $\gamma(.; B): S \to R$ is continuously differentiable, or C^1, for all closed events $B \epsilon \mathcal{B}(S \times P)$, and $\gamma(s^t; \cdot) \epsilon \mathcal{M}(S \times P)$ for all $s^t \epsilon S$. Moreover, the support of $\gamma(s^t; .)$ is contained in a fixed compact subset of S x P for each $s^t \epsilon S$.[3]

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[3] For each $s^t \epsilon S$, the support of a probability measure $\gamma(s^t; \cdot)$ is the smallest closed set in S x P with full measure, and it is well defined (see Parthasarathy (1967)). The second part of Assumption 1 is standard in the temporary equilibrium analysis, named "inelasticity" of expectations (see, for instance, Grandmont (1974)).

We recall that $\gamma(s^t; B)$ is continuously differentiable at $s^t \in S$ if $\gamma(s^t; B)$ is continuous at s^t and there is a continuous linear map $D_s\gamma(s^t; B)$: $T_s(S) \to R$ such that $D_s\gamma(s^t; B)(h) = \lim_{\alpha\to 0} \frac{1}{\alpha}[\gamma(s^t+\alpha h; B) - \gamma(s^t; B)]$ for each $h \in T_s(S)$, where $T_s(S)$ is the tangent space of S at s^t. Further, $\gamma(\cdot;B)$ is continuously differentiable if $\gamma(s^t; B)$ is continuously differentiable at all $s^t \in S$. Since $\gamma(s^t;\cdot) \in M(S \times P)$, it can be seen easily that the derivative $D_s\gamma(s^t;\cdot)$ is itself a (finite signed) measure for each $s^t \in S$. First, it is clear that $-\infty < D_s\gamma(s^t; S \times P) < \infty$ by definition of a derivative. Moreover, $D_s\gamma(s^t; \phi) = 0$ since $\gamma(s^t; \phi) = 0$ for all $s^t \in S$, and $D_s\gamma(s^t; \bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} D_s\gamma(s^t; E_i)$ for any sequence $\{E_i\}$ of disjoint measurable sets in SxP. The latter is because that $\gamma(s^t; \bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} \gamma(s^t; E_i)$ for any sequence $\{E_i\}$ of disjoint measurable sets in SxP, and then for each $h \in T_s(S)$,

$$\begin{aligned}
D_s\gamma(s^t; \bigcup_{i=1}^{\infty} E_i)(h) &= \lim_{\alpha\to 0} \frac{1}{\alpha}\Big[\gamma(s^t+\alpha h; \bigcup_{i=1}^{\infty} E_i) - \gamma(s^t; \bigcup_{i=1}^{\infty} E_i)\Big] \\
&= \lim_{\alpha\to 0} \frac{1}{\alpha}\Big[\sum_{i=1}^{\infty}\gamma(s^t+\alpha h; E_i) - \sum_{i=1}^{\infty}\gamma(s^t; E_i)\Big] \\
&= \sum_{i=1}^{\infty} \lim_{\alpha\to 0} \frac{1}{\alpha}\big[\gamma(s^t+\alpha h; E_i) - \gamma(s^t; E_i)\big] \\
&= \sum_{i=1}^{\infty} D_s\gamma(s^t; E_i)(h) \quad .
\end{aligned}$$

As usual, we further assume that the consumer's intertemporal preferences among consequences (satisfying Expected Utility Hypothesis) can be represented by a von Neumann-Morgenstern utility function u satisfying the following:

<u>Assumption 2</u>: $u: P \times P \to R$ is bounded and it is of class C^k with $k > 2$. Further, $D_2u(x^t, x^{t+1}) >> 0$ for every $(x^t, x^{t+1}) \in P \times P$,

where $D_2u(x^t, x^{t+1})$ is the gradient vector of $u(x^t, x^{t+1})$ with respect to x^{t+1}. Suppose the consumer has taken an action $(x^t, m^t) \in P \times R_+$ at time t and faces a future environment $(s^{t+1}, \bar{x}^{t+1}) \in S \times P$ for each $s^t \in S$. Then his decision problem in period t for period t+1 is to choose x^{t+1} satisfying $p^{t+1} \cdot x^{t+1} = p^{t+1} \cdot \bar{x}^{t+1} + m^t$ and that $u(x^t, x^{t+1})$ is maximized. Hence, a plan is a correspondence defined by

$$\hat{x}^{t+1}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) = \{x^{t+1} \in P: u(x^t, x^{t+1}) \text{ is maximized and } p^{t+1} \cdot x^{t+1} = p^{t+1} \cdot \bar{x}^{t+1} + m^t\}$$

for each $(x^t, m^t) \in P \times R_+$ and $(s^{t+1}, \bar{x}^{t+1}) \in S \times P$. Let $\hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) = u(x^t, \hat{x}^{t+1}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}))$. In order to preserve enough differentiability for the analysis, a classical assumption on the future market is made as follows:

Assumption 3: $D_2^2u(x^t, x^{t+1})$ is negative definite on the space $\{\mu \in R^\ell: D_2u(x^t, x^{t+1}) \cdot \mu = 0\}$ for every $(x^t, x^{t+1}) \in P \times P$,

where $D_2^2u(x^t, x^{t+1})$ is the bilinear symmetric form of $u(x^t, x^{t+1})$. Assumption 3 means that the "Hessian" is negative definite for small disturbance along the contour of u with respect to the future consumption. Or, given $x^t \in P$, the Gaussian curvature at x^{t+1} of the indifference hypersurface through x^{t+1} is different from zero (see Debreu (1972)). In summary, the consumer is risk averse with respect to the future consumption.

If s^t is the price system at time t, the consumer's expected utility function of an action (x^t, m^t) is defined by

$$v(x^t, m^t, s^t) = \int_{S \times P} \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) \gamma(s^t; d(s^{t+1}, \bar{x}^{t+1}))$$

$$= \int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})\gamma(s^t; d(s^{t+1}, \bar{x}^{t+1})),$$

where K is the common compact subset of S x P which contains the support of $\gamma(s^t;\cdot)$ for all $s^t \in S$, according to Assumption 1. It is clear that $v(x^t,m^t,s^t) = \hat{u}(x^t, m^t, s^t)$ if γ is a point expectation, or equivalently there is no uncertainty involved in the model. We observe that the expected utility function v defined above depends on money m^t and current price s^t explicitly, which reflects a "generalized real balance effect" (see, for example, Dusansky and Kalman (1972)). It is important to notice that, in general, v is not homogeneous of any degree in m^t and s^t, thus we allow for the possibility of "money illusion" in the expected utility function v (see, for example, Dusansky and Kalman (1974) and Grandmont (1974)). We have the following:

Main Theorem: The expected utility function $v: P \times R_+ \times S \to R$ is C^1. In particular, $v(.,.,s^t): P \times R_+ \to R$ is C^{k-1} with $k > 2$ for every $s^t \in S$.[4]

Now the consumer's decision problem in period t can be stated as follows: if s^t is quoted in period t, the consumer facing a future environment $(s^{t+1}, \bar{x}^{t+1})$ will choose an action (x^t, m^t) to maximize the expected utility $v(\cdot,\cdot, s^t)$ over the budget condition $p^t\cdot x^t + m^t = p^t\cdot\bar{x}^t + m^{t-1}$ provided that the plan for the future consumption $\hat{x}^{t+1}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})$ is realized.

The rest of this note devotes to present the detail of the proof of the main theorem. The tools are the following two lemmas:

Lemma 1 (Inverse function theorem (Milnor (1965)):

Let $f: R^n \to R^n$ be of class C^k and $Df(x)$ has rank n at $x \in R^n$. Then f is a diffeomorphism of class C^k of a neighborhood of x onto a neighborhood in R^n of $f(x)$.

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[4] A function defined on a closed subset X of R^n is said to be of class C^k if and only if there exists an open set U containing X and a C^k-extension of f to U.

Lemma 2 (Implicit function theorem (Dieudonne (1969)):

Let $f: R^n \times R^m \to R^m$ be of class C^k in a neighborhood containing (x,y) where $x \in R^n$, $y \in R^m$, and $f(x,y) = 0$. If $D_y f(x,y)$ is nonsingular, then there is a unique function g mapping a neighborhood of $x \in R^n$ into R^m such that $g(x) = y$ and $f(x', g(x')) = 0$ for x' in this neighborhood. Moreover, g is of class C^k.

By definition, given an action $(x^t, m^t) \in P \times R_+$ and a future event $(s^{t+1}, \bar{x}^{t+1}) \in S \times P$, the consumption plan can be rewritten as follows:

$$\hat{x}^{t+1}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})$$

$$= \{x^{t+1} \in P: D_2 u^h(x^t, x^{t+1}) = \lambda \cdot p^{t+1} \text{ and }$$

$$p^{t+1} \cdot x^{t+1} = p^{t+1} \cdot \bar{x}^{t+1} + m^t\}.$$

where λ is the Lagrangian multiplier. Let $\alpha(x^{t+1}, \lambda | x^t, m^t; s^{t+1}, \bar{x}^{t+1}) = (D_2 u(x^t, x^{t+1}) - \lambda \cdot p^{t+1}, p^{t+1} \cdot \bar{x}^{t+1} + m^t - p^{t+1} \cdot x^{t+1})$. Obviously, α is C^{k-1} on $P \times R \times P \times R_+ \times S \times P$. We claim that the derivative matrix of $\alpha(x^{t+1}, \lambda | x^t, m^t; s^{t+1}, \bar{x}^{t+1})$ with respect to (x^{t+1}, λ), denoted by $D_{(x^{t+1}, \lambda)} \alpha(x^{t+1}, \lambda | x^t, m^t; s^{t+1}, \bar{x}^{t+1})$ has full rank for each $(x^t, m^t; s^{t+1}, x^{t+1}) \in P \times R_+ \times S \times P$. This follows from Assumption 3, that is, for every $(x^t, x^{t+1}) \in P \times P$,

$$\det (D_{(x^{t+1},\lambda)} \alpha(x^{t+1}, \lambda | x^t, m^t; s^{t+1}, \bar{x}^{t+1}))$$

$$= \det \begin{pmatrix} D_2^2 u(x^t, x^{t+1}) & -p^{t+1} \\ & \\ -p^{t+1} & 0 \end{pmatrix} \neq 0$$

(Note that without causing confusion we use the same vector notation for the column and row vectors). Hence $\alpha(\cdot,\cdot|x^t, m^t; s^{t+1}, \bar{x}^{t+1})$ is a local C^1 diffeomorphism for any $(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) \epsilon P \times R_+ \times S \times P$ by Lemma 1. Let $\alpha(x^{t+1}, \lambda|x^t, m^t; s^{t+1}, \bar{x}^{t+1}) = 0$. Then, by applying Lemma 2, there is a unique C^{k-1} function f from a neighborhood of $(x^t, m^t; s^{t+1}, x^{t+1})$ in $P \times R_+ \times S \times P$ into $P \times R$ with $k > 2$ such that $f(x^t, m^t; s^{t+1}, x^{t+1}) = (x^{t+1}, \lambda)$ and $\alpha(f(x^{t'}, m^{t'}; s^{t+1'}, \bar{x}^{t+1'})|x^{t'}, m^{t'}; s^{t+1'}, x^{t+1'}) = 0$ for any $(x^{t'}, m^{t'}; s^{t+1'}, \bar{x}^{t+1'})$ in this neighborhood. Since f is unique, set f = (x^{t+1}, λ). This proves that $\hat{x}^{t+1}$ is C^{k-1} on $P \times R_+ \times S \times P$, and hence the utility function $\hat{u}$ is of class C^{k-1}. Further, since the support of $\gamma(s^t;\cdot)$ is contained in a fixed compact subset K of $S \times P$ for each $s^t \epsilon S$ (Assumption 1) and $\hat{u}$ is C^{k-1} as shown above, then by Leibniz rule ([4], p. 177), $v(\cdot,\cdot, s^t)$ is C^{k-1} on $P \times R_+$ with $k > 2$ for each $s^t \epsilon S$, and

$$D^\beta_{(x,m)} v(x^t, m^t, s^t) = \int_K D^\beta_{(x,m)} \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) \gamma(s^t; d(s^{t+1}, \bar{x}^{t+1}))$$

for each $(x^t, m^t) \epsilon P \times R_+$ and $\beta = 0,1,\ldots,k-1$. Now, to show v is of class C^1, we have to prove (i) $v(x^t, m^t,\cdot)$ is continuous on S for each $(x^t, m^t) \epsilon P \times R_+$, and (ii) the derivative $D_s v(x^t, m^t, s^t)$ exists and it is continuous for all $s^t \epsilon S$. By definition, for each $h \epsilon T_s(S)$,

$$\begin{aligned} D_s v(x^t, m^t, s^t)(h) &= \lim_{\alpha\to 0} \frac{1}{\alpha} [v(x^t, m^t, s^t+\alpha h) - v(x^t, m^t, s^t)] \\ &= \lim_{\alpha\to 0} \frac{1}{\alpha} [\int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) \gamma(s^t+\alpha h; d(s^{t+1}, \bar{x}^{t+1})) \\ &\quad - \int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1}) \gamma(s^t; d(s^{t+1}, \bar{x}^{t+1}))]. \end{aligned}$$

We recall that the difference of two probability measures is a signed measure, so is the nonzero multiple of a probability measures. Hence, for each $h \epsilon T_s(S)$,

let $\gamma_\alpha^h(s^t;\cdot) = \frac{1}{\alpha}[\gamma(s^t+\alpha h;\cdot) - \gamma(s^t;\cdot)]$. $\gamma_\alpha^h(s^t;\cdot)$ is a finite signed measure since $\alpha \neq 0$, and it is continuous with respect to α because $\gamma(\cdot;B)$ is of class C^1 for each closed event $B \in \mathcal{B}(S \times P)$. Therefore, as α approaches to 0, $\gamma_\alpha^h(s^t;\cdot)$ converges weakly to $\gamma_o^h(s^t;\cdot) = D_s\gamma(s^t;\cdot)(h)$. That is, for all continuous bounded functionsf: $S \times P \to R$, $\int f(z)\gamma_\alpha^h(s^t;dz)$ converges to $\int f(z)\gamma_o^h(s^t;dz) = \int f(z)D_s\gamma(s^t;dz)(h)$. Hence for each $h \in T(S)$,

$$D_s v(x^t, m^t, s^t)(h) = \lim_{\alpha \to 0} \int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})\gamma_\alpha^h(s^t; d(s^{t+1}, \bar{x}^{t+1}))$$

$$= \int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})\gamma_o^h(s^t; d(s^{t+1}, \bar{x}^{t+1}))$$

$$= \int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})D_s\gamma(s^t; d(s^{t+1}, \bar{x}^{t+1}))(h)$$

Since $D_s\gamma(s^t;\cdot)$ exists and it is a (finite signed) measure, and $\hat{u}(x^t, m^t;\cdot,\cdot)$ is of class C^{k-1} with $k > 2$, therefore $D_s v(x^t, m^t, s^t)$ exists and it is well defined for each $(x^t, m^t, s^t) \in P \times R_+ \times S$. To show the continuity of $v(x^t, m^t,\cdot)$ and $D_s v(x^t, m^t,\cdot)$ on S for each $(x^t, m^t) \in P \times R_+$, consider the sequence $\{s_n^t\}$ in S converging to s_o^t. We know that $\gamma(\cdot;B)$ and $D_s\gamma(\cdot;B)$ are continuous for all closed events $B \in \mathcal{B}(S \times P)$, then the sequences $\{\gamma(s_n^t;\cdot)\}$ and $\{D_s\gamma(s_n^t;\cdot)\}$ converge weakly to $\gamma(s_o^t;\cdot)$ and $D_s\gamma(s_o^t;\cdot)$, respectively. That is, $\int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})\gamma(s_n^t;d(s^{t+1}, \bar{x}^{t+1}))$ and $\int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})D_s\gamma(s_n^t; d(s^{t+1}, \bar{x}^{t+1}))$ converge to $\int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})$ $\gamma(s_o^t; d(s^{t+1}, \bar{x}^{t+1}))$ and $\int_K \hat{u}(x^t, m^t; s^{t+1}, \bar{x}^{t+1})D_s\gamma(s_o^t; d(s^{t+1}, \bar{x}^{t+1}))$, respectively (compare with Grandmont (1972), Section 5, Theorem A.3). This completes the proof of the theorem.

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13. ABSTRACT

In the setting of a simple two-period model with money, the continuous differentiability of the expected utility function is shown to follow from the following assumptions provided the monotone von Neumann-Morgenstern utility function and the expectation function are sufficiently differentiable: (a) inelasticity of expectations; (b) risk aversion in future consumption.

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